Space charge limited current flow between coaxial cylinders at potentials up to 15 MV

1977 J. Phys. A: Math. Gen. 10 631

(http://iopscience.iop.org/0305-4470/10/4/024)

J. Phys. A: Math. Gen., Vol. 10, No. 4, 1977. Printed in Great Britain. © 1977

Space charge limited current flow between coaxial cylinders at potentials up to 15 MV

C B Wheeler
Plasma Physics Department, Imperial College, London SW7 2AZ, UK

Received 3 December 1976

Abstract. Poisson's equation in cylindrical symmetry is solved numerically with the assumptions of zero field and zero electron energy at the cathode surface. The computed current is expressed in terms of an analytical solution obtained for the extreme relativistic case. Internal and external anode configurations are treated, with ratio of electrode radii extending to 10 000 and potential difference up to 15 MV. Application of this analysis to the high-power field-emission diode is considered, with particular reference to the radially pumped ultraviolet laser.

1. Introduction

Many of the experimental approaches to controlled thermonuclear fusion require, at one stage or another, energy input in the form of a high-density beam of relativistic electrons. As these experiments become more sophisticated there is a demand for greater electron densities and energies. A brief review of the experimental installations presented in a paper by Wallis *et al* (1975) shows that energies up to 15 MV and beam currents over 1MA are presently available. In all cases the electrons are liberated by field emission at the cathode surface and the electrode configuration is chosen to suit the beam geometry required. Frequently this configuration approximates to a portion of a pair of parallel infinite planes, or coaxial cylinders, or concentric spheres. One particular situation which closely realizes the cylindrical configuration is the coaxial form of ultraviolet laser introduced by Bradley *et al* (1974). In this device the anode is a thin-walled cylinder at earth potential that contains the gaseous lasing medium. Application of a high-voltage pulse to the surrounding coaxial cathode produces a radially converging beam of electrons that acquires sufficient energy to penetrate the anode wall and to excite the contained gas. This technique results in very uniform pumping and the close coupling gives a high efficiency.

The demand for higher electron energy has prompted many authors to extend the early Child–Langmuir analysis of the planar diode into the relativistic régime. All the treatments assume that the current is not limited by the rate of electron production at the cathode surface. Acton (1957), Howes (1966), Boers and Kelleher (1969), Godyak *et al* (1970), assume zero cathode emission energy and evaluate the space charge limited current. This is the maximum current that can flow in the diode at a given potential difference and occurs when space charge effects reduce the field at the cathode surface to zero. Jory and Trivelpiece (1969) include the effect of a finite cathode field and Wheeler (1974) additionally includes a cathode emission velocity.

The problem of current flow between coaxial cylinders has received little attention since the original non-relativistic treatments by Langmuir (1913), Langmuir and Blodgett (1923) and Langmuir and Compton (1931). Approximate analytic expressions for the space charge limited current were derived by Acton (1957) for certain ranges of potential in the relativistic régime, but these ranges are not well defined and do not overlap. Wheeler (1975) extended the early non-relativistic analysis to include all values of the cathode field. The computations presented here evaluate the space charge limited current flowing between a pair of coaxial cylinders of either polarity for potentials up to 15 MV, assuming zero cathode emission energy. Since these calculations are primarily intended for application to high-power field emission diodes, it is important to justify the theoretical assumptions in relation to these devices. Wheeler (1973, 1974) has shown that inclusion of the field-emission energy spectrum into the theory has a negligible effect on the diode current. However the assumption of zero cathode field would appear to be completely invalid since the Fowler–Nordheim theory indicates that cathode fields of between 10^7 and 10^8 V cm^{-1} are required for significant electron emission from metallic surfaces with work functions between 4 and 5 V. High-power field-emission diodes are constructed with very roughened cathode surfaces so that fields of this order are obtained locally in the initial stages of operation, before space charge effects dominate. High-speed photographic observation shows that a plasma is formed at the cathode surface as soon as the diode conducts, presumably due to interaction between field-emission electrons and expelled material. The physical processes taking place are unclear but it is generally accepted that this plasma subsequently acts as the cathode surface. Such a plasma cathode has a very low work function and therefore permits a large emission current even when the surface field is reduced to a low value by space charge effects. Support for this cathode interpretation is provided by the experimental observations of Clark *et al* (1969). These authors tabulate the measured impedances of many high-power field-emission diodes in use at that time. For the diodes with a near planar geometry the inferred experimental currents are less than a factor of two different from the theoretical space charge limited value.

2. Mathematical formulation

Consider a coaxial cylindrical geometry of infinite length comprising an anode cylinder, of radius r_1, maintained at a potential V_1 with respect to the cathode cylinder, radius r_0. In the steady state the potential and space charge density ρ in the inter-electrode region are related through Poisson's equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}V}{\mathrm{d}r}\right) = -4\pi\rho. \tag{1}$$

If the electrons are emitted from the cathode with zero energy then ρ can be related to the current I per unit electrode length and also to the electron velocity v which, in turn, can be related to the local potential

$$I = -2\pi r\rho v, \qquad mc^2(1-v^2/c^2)^{-1/2} = mc^2 + eV. \tag{2}$$

These equations assume that the current I is sufficiently low for self-magnetic fields not to influence the electron motion. It is convenient to express radii and potentials in the following non-dimensional form:

$$x = \ln(r/r_0), \qquad y = eV/mc^2. \tag{3}$$

Equations (1), (2) and (3) combine to give

$$d^2y/dx^2 = \tfrac{1}{2}A\,e^x(1+y)(y^2+2y)^{-1/2} \tag{4}$$

where $A = 4er_0I/mc^3$. In order to evaluate the space charge limited current this equation must be solved subject to zero field at the cathode surface, i.e. to $(dy/dx)_0 = 0$, assuming that the emission process causes no limitation. An analytic solution is possible in the following two simple cases. Firstly, if $x \ll 1$ and $y \ll 1$, i.e. $|r - r_0| \ll r_0$ and $eV \ll mc^2$, then the right-hand side of equation (4) has a simple $y^{-1/2}$ dependence and double integration gives

$$x = 2^{7/4}3^{-1}A^{-1/2}y^{3/4} \qquad x \ll 1, y \ll 1. \tag{5}$$

This is simply the Child–Langmuir relation for the current density, $I/2\pi r_0$, of non-relativistic electrons flowing between infinite plane electrodes separated by a distance $|r - r_0|$. Secondly if $y \gg 1$ for all x then the right-hand side of equation (4) is independent of y and double integration gives

$$A = 2y(e^x - x - 1)^{-1} \qquad y \gg 1. \tag{6}$$

This equation assumes that the electrons move with the velocity of light over the entire inter-electrode region, including the cathode surface. In terms of anode radius and anode potential this extreme relativistic approximation yields a current

$$I_r = \frac{cV_1}{2r_0}[(r_1/r_0) - \ln(r_1/r_0) - 1]^{-1}. \tag{7}$$

Equation (4) can be expressed in a form suitable for general numerical solution by multiplying throughout by $2dy/dx$ and integrating twice with respect to y:

$$x_1 = A^{-1/2}\int_0^{y_1}\left(\int_0^{y} e^x(1+y)(y^2+2y)^{-1/2}\,dy\right)^{-1/2} dy. \tag{8}$$

This equation is integrated numerically, from the cathode towards the anode, using the non-relativistic planar approximation of equation (5) as a first approximation for x in the integrand. The improved value of x so obtained is then used as a second approximation, and so on.

3. Results and discussion

For a prescribed value of A, i.e. specified r_0I, the numerical integration yields sets of coordinates (x_1, y_1), i.e. coordinates $(r_1/r_0, V_1)$. For both external and internal anode, i.e. positive and negative x, solution for $10^{-3} \leqslant A \leqslant 10^2$ provides sufficient coordinates to cover a range of up to 10 000 in ratio of electrode radii. The most concise way of presenting these results is in the form of a numerical correction factor to be applied to the extreme relativistic approximation of equation (7). The parameter presented is the ratio I_r/I which always exceeds unity since the approximation I_r assumes an electron velocity c at the cathode whereas the current I is calculated for zero emission energy. Figures 1(a) and 1(b) show the results for an external anode with potential differences ranging from 0·5 MV to 15·0 MV. The ratio of radii is presented in multiples of $\sqrt{(10)}$, which is convenient for interpolation to any value in the range $1 \leq r_1/r_0 \leq 10\ 000$. The corresponding results for an internal anode are shown in figures 2(a) and 2(b). In both configurations the current I naturally approaches the extreme relativistic value I_r as the potential difference increases. However, the approach is most rapid when the cathode curvature is greatest ($r_1/r_0 \to \infty$ for $r_1 > r_0$ or $r_0/r_1 \to 1$ for $r_1 < r_0$) since this is the

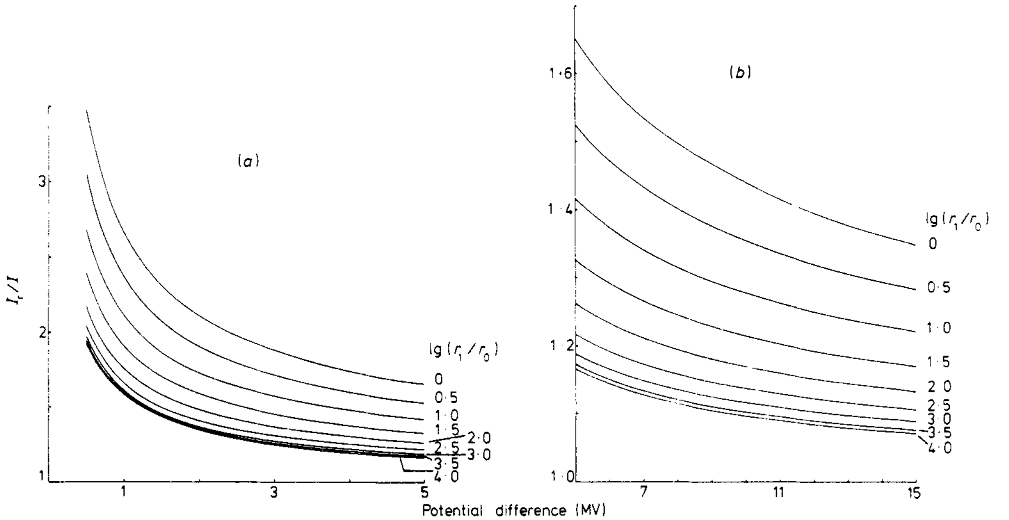

Figure 1. External anode. Calculated current I in terms of the approximation I_r of equation (7) as a function of voltage and electrode radii. (a) 0·5 MV to 5 MV; (b) 5 MV to 15 MV.

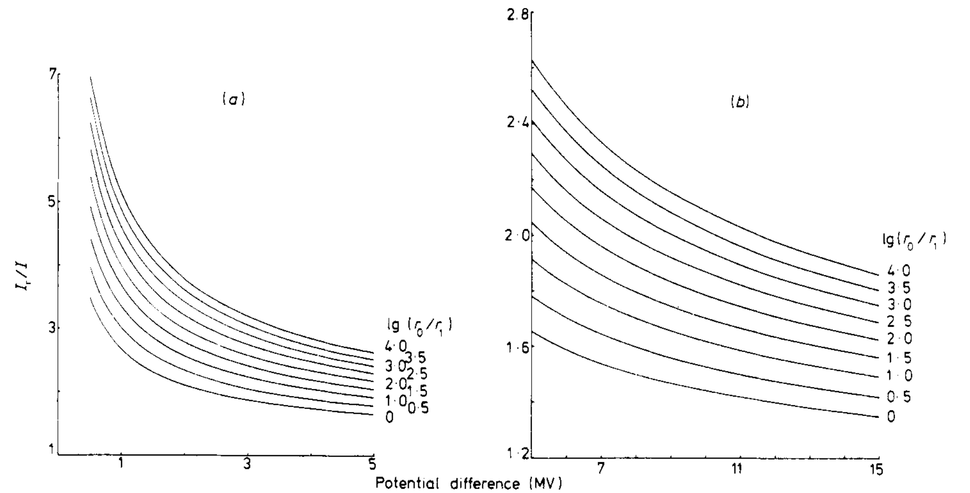

Figure 2. Internal anode. Calculated current I in terms of the approximation I_r of equation (7) as a function of voltage and electrode radii. (a) 0·5 MV to 5 MV; (b) 5 MV to 15 MV.

condition that maximizes the acceleration of the electrons at the beginning of their traverse. At the lowest potential presented, 0·5 MV, the non-relativistic calculations of Langmuir and Blodgett (1923) can considerably overestimate the current. This discrepancy amounts to 10% for comparable radii ($r_1/r_0 \rightarrow 1$), decreases for the internal anode as r_0/r_1 increases and increases for the external anode as r_1/r_0 increases. In this latter configuration the discrepancy levels off at 22% for r_1/r_0 in excess of 100.

Lasers operating in the far ultraviolet are currently pumped by coaxial diodes of radii $1 < r_0/r_1 < 10$, operated at potentials up to about 1 MV. At higher potentials the range of the electrons in the gaseous lasing medium is too great and the pumping efficiency falls off. However, in the quest for greater lasing outputs it is likely that gas pressures will be raised in the future and possibly the liquid state used. This will necessitate pumping by electrons of greater energy and for this application figure 3

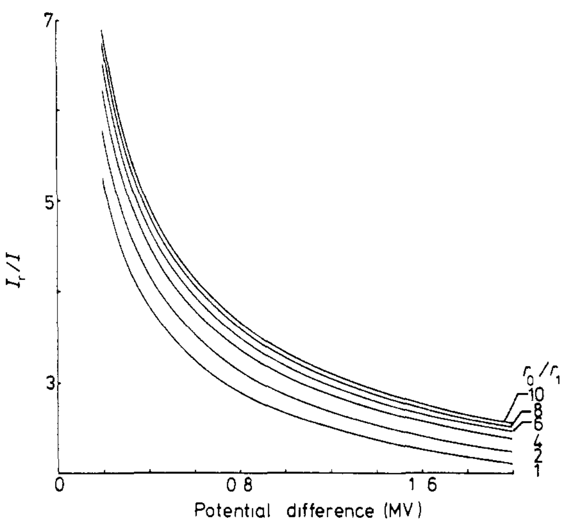

Figure 3. Internal anode. Calculated current I in terms of the approximation I_r of equation (7) as a function of voltage, 0·2 MV to 2 MV, and electrode radii.

shows the currents obtainable from such diodes when operated in the potential range 0·2 MV to 2·0 MV. At the lower potential the non-relativistic calculations of Langmuir and Blodgett (1923) overestimate the current by less than 4% over the range of radii presented. Operation at high current densities can cause the path of the electrons in the vicinity of the anode to depart from the purely radial direction, due to the influence of the magnetic field generated by the current flowing axially in the anode structure. As a result the electron beam tends to pinch, leading to non-uniform pumping of the lasing medium which produces a laser beam of poor divergence. The continuing search for lasing transitions at even shorter wavelengths will, however, necessitate pumping at these high current densities. Schlitt and Bradley (1975) discuss quantitatively how this beam pinching can be reduced by diode segmentation in the axial direction and their analysis uses an empirical formula for the space charge limited diode current. It should be possible to obtain better design parameters by embodying the present accurate calculations of diode current into their theory.

References

Acton E W V 1957 *J. Electron.* **3** 203–10
Boers J E and Kelleher D 1969 *J. Appl. Phys.* **40** 2409–12
Bradley D J, Hull D R, Hutchinson M H R and McGeoch M W 1974 *Opt. Commun.* **11** 335–8
Clark J J, Ury M, Andrews M L, Hammer D A and Linke S 1969 *10th Symp. on Electron, Ion and Laser Beam Technology* (San Francisco: San Francisco Press) pp 117–30
Godyak V A, Dubovoi L B and Zablotskaya G R 1970 *Sov. Phys.-JETP* **30** 971–2
Howes W L 1966 *J. Appl. Phys.* **37** 438–9
Jory H R and Trivelpiece A W 1969 *J. Appl. Phys.* **40** 3924–6
Langmuir I 1913 *Phys. Rev.* **2** 450–86
Langmuir I and Blodgett K B 1923 *Phys. Rev.* **22** 347–56
Langmuir I and Compton K T 1931 *Rev. Mod. Phys.* **3** 191–257
Schlitt L G and Bradley L P 1975 *Proc. Int. Conf. on Electron Beam Research and Technology, Albuquerque, New Mexico* vol. 2, ed. G Yonas (Springfield, Virginia: US Department of Commerce)
Wallis G, Sauer K, Sunder D, Rosinskii S E, Rukhadze A A and Rukhlin V G 1975 *Sov. Phys.–Usp.* **17** 492–506
Wheeler C B 1973 *J. Phys. A: Math., Nucl. Gen.* **6** 1439–50
—— 1974 *J. Phys. D: Appl. Phys.* **7** 267–79
—— 1975 *J. Phys. A: Math. Gen.* **8** 555–9